Onsager's conjecture for general conservation laws

The studies on conservation of energy for incompressible Euler system are a part of the whole programme, which was initiated in the nineties and the first two decades of the current century are a real explosions of results which are consequent breakthroughs in the theory. This programme concentrated around the Onsager conjecture with its both directions - conservation of energy and existence of solutions that would dissipate the energy in some anomalous way and search for intimate relation of this theory with turbulence, which has always been a challenge both for physicists and mathematicians. These discoveries for some time seemed to be related only with very particular system - incompressible Euler system, having only bilinear nonlinearity. This was a common feeling of many experts.

Our recent findings for general hyperbolic systems gave a new light for the whole story. Apparently the Onsager-like statement in its positive direction became true not only for systems similar to incompressible Euler with any bilinear nonlinearity, like e.g. incompressible magnetohydrodynamics, but also for compressible models, and most importantly for general hyperbolic systems. An emerging question became whether this whole programme, that was believed to work for incompressible Euler extends in fact to more general equations - to hyperbolic (or not necessarily hyperbolic) conservation laws? The question of regularity requirement of weak solutions to satisfy the entropy equality, or in other words additional companion law is just one of the elements. Still what is waiting for an answer are the questions of solutions being continuous that would not satisfy companion law and understanding how this is all related to turbulence. The aim of this project is to contribute in a significant way to answering these new appearing questions.

As the negative direction of the Onsager conjecture states, nonconservation of energy in a turbulent flow might occur not only from viscous dissipation, but also from lack of smoothness of the velocity. The question of existence of Hölder regular flows has driven the development of the method of convex integration in fluid mechanics – De Lellis and Székelyhidi constructed initial data leading to solutions not preserving the energy. These results used completely different framework to standard methods used for partial differential equations, namely the method of convex integration. This construction reached to the methods of differential geometry, and in particular to the celebrated Nash-Kuiper theorem. In analogue to Nash's construction, in fluid dynamics in each step of iteration a subsolution is produced from the previous one by adding a perturbation, which is a highly oscillatory sequence.

Due to the well recognized dominant role of the boundary in the generation of turbulence it seems very reasonable to investigate the analogue of the Onsager conjecture in bounded domains, which is one of the aims of the project. Eventually, the need to localize in order to deal with the boundary effect stimulates the construction of a direct proof which may have further applications.